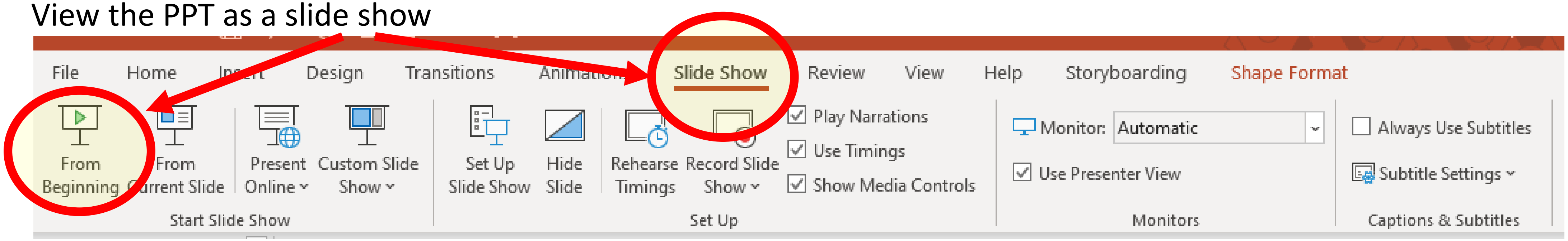
How to best use these slides…

- View the PPT as a slide show



- Then click through every step
    - Mouse clicks will advance the slide show
    - Left/right arrow keys move forward/backward
    - Mouse wheel scrolling moves forward/backward
- When a question is posed, stop and think it through, try to answer it yourself before clicking
- If you have questions, email me, ask in the Teams Student Center channel!

LESSON 6.5b

Change of Base Formula for Logarithms

Today you will:

- Learn how to use the “Change of Base” Formula for logarithms to solve “unusual” log problems.
- Practice using English to describe math processes and equations

Core Vocabulary:

- Change-of-Base Formula, p. 329

Previous:

- Base of an exponent and of a logarithm

Logarithm

Another way of writing $y = b^x$

$$\log_b y = x$$

Read it as "Log base b of y is x"

…as an exponential function it is b to the x is y

You know how to use your calculator to solve the following:

$\log 8$

or

$\ln 0.3$

…use the "log" and "ln" buttons on the far left of your TI calculator…

```
log(8)
          .903089987
ln(0.3)
        -1.203972804
```

…but what about something like:

$\log_{13} 1.35$

there is no button for logarithm using any base other 10 or e

…what to do?

…what to do?

Change-of-Base Formula for Logarithms

$$\log_c a = \frac{\log_b a}{\log_b c}$$

Pick your base!

What would be some good bases to use?

base 10
or
base *e*

they're on your calculator!

ANOTHER WAY

In Example 4, $\log_3 8$ can be evaluated using natural logarithms.

$\log_3 8 = \frac{\ln 8}{\ln 3} \approx 1.893$

Notice that you get the same answer whether you use natural logarithms or common logarithms in the change-of-base formula.

Evaluate $\log_3 8$ using common logarithms.

SOLUTION

$$\log_3 8 = \frac{\log 8}{\log 3} \qquad \log_c a = \frac{\log a}{\log c}$$

$$\approx \frac{0.9031}{0.4771} \approx 1.893 \qquad \text{Use a calculator. Then divide.}$$

Evaluate $\log_6 24$ using natural logarithms.

SOLUTION

$$\log_6 24 = \frac{\ln 24}{\ln 6}$$

$$\log_c a = \frac{\ln a}{\ln c}$$

$$\approx \frac{3.1781}{1.7918} \approx 1.774$$

Use a calculator. Then divide.

For a sound with intensity I (in watts per square meter), the loudness $L(I)$ of the sound (in decibels) is given by the function

$$L(I) = 10 \log \frac{I}{I_0}$$

where I_0 is the intensity of a barely audible sound (about 10^{-12} watts per square meter). An artist in a recording studio turns up the volume of a track so that the intensity of the sound doubles. By how many decibels does the loudness increase?

SOLUTION

Let I be the original intensity, so that $2I$ is the doubled intensity.

$$\begin{aligned}
\text{increase in loudness} &= L(2I) - L(I) && \text{Write an expression.} \\
&= 10 \log \frac{2I}{I_0} - 10 \log \frac{I}{I_0} && \text{Substitute.} \\
&= 10 \left(\log \frac{2I}{I_0} - \log \frac{I}{I_0} \right) && \text{Distributive Property} \\
&= 10 \left(\log 2 + \log \frac{I}{I_0} - \log \frac{I}{I_0} \right) && \text{Product Property} \\
&= 10 \log 2 && \text{Simplify.}
\end{aligned}$$

The loudness increases by $10 \log 2$ decibels, or about 3 decibels.

REVIEW: Change-of-Base Formula for Logarithms

$$\log_c a = \frac{\log_b a}{\log_b c}$$

Pick your base!

What would be some good bases to use?

base 10
or
base *e*

they're on your calculator!

Homework

Pg 332, #33-45